INFORMATION
TECHNOLOGY
UNIVERSITY

CRACKING THE CODING INTERVIEW

Master Problem Solving for Technical Interviews

S_1 $\{1, 2, 3, 4, 5, \dots, n\} \Rightarrow |S_1| = n$

S_2 $\{2, 4, 6, 8, 10, \dots, n\} \Rightarrow |S_2| = \frac{n}{2}$

S_3 $\{1, 6, 11, \dots, n\} \Rightarrow |S_3| = \frac{n}{5}$

S_4 $\{1, k, 2k, \dots, n\} \Rightarrow |S_4| = \frac{n}{k}$

k, 2k, 3k
... so take
floor (n/k)

S_5 $\{0, \sqrt{n}, 2\sqrt{n}, \dots, n\} \Rightarrow |S_5| = \frac{n}{\sqrt{n}} = \boxed{\sqrt{n}}$

$for\ (i = 1, i \leq n, i{+}{+})$

$k = n$

$for\ (i = 1;\ i \leq n;\ i = i + \boxed{k})$

$for\ (i = 0;\ \boxed{i < n};\ i = i + \sqrt{n}) \Rightarrow \sqrt{n}$

i step $\boxed{\sqrt{n}}$

- $i = 0$
- $i = \sqrt{n}$
- $i = 2\sqrt{n}$
- $\dots$
- $i < n$

$\frac{n}{\sqrt{n}} = \sqrt{n}$ steps $\approx n$

$\frac{\sqrt{n}}{\sqrt{n}} = 1$ $\quad \frac{\sqrt{n}}{\sqrt{n}}$

$t + 1 < n \Rightarrow t \approx \sqrt{n}$

$for\ (i = 1;\ i \leq n;\ i = i \boxed{+2})$

$1,\ \underbrace{3,\ 5,}_{2\sqrt{n}}\ 7,\ \dots$

$\underbrace{\frac{n}{2},\ \frac{n}{2} + 1,\ \frac{n}{2} + 2,}_{\approx \frac{n}{2}}\ \dots,\ n$

$\therefore \approx \frac{n}{2}$

$for\ (i = 1;\ i \leq n;\ i = \boxed{i \times 2}) \Rightarrow (n)^{1/2}$

$1,\ 2,\ 4,\ 8,\ 16,\ \dots,\ \leq n$

$\log_2 n$

$\Rightarrow n^{1/2},\ n^{1/4},\ n^{1/8},\ \dots,\ n^{\frac{1}{2^k}}$

$n^{\frac{1}{2^k}} \leq 1 \Rightarrow 2^k \geq \log_2 n$ $\quad k \approx \log_2 n$

$$\boxed{(a^x)^y = a^{x \cdot y}}$$

Arithmetic: $A \cdot d \Rightarrow n/d \Rightarrow \boxed{\dfrac{n \cdot a_1}{d}}$

$$1 + 2 + 3 + 4 + 5 + 6 + \dots + (n-3) + (n-2) + (n-1) + n$$

```
Sum = 0
for ( i = 1 ; i <= n ; i++ )
    Sum += i
```

$\Rightarrow$ Sum ???

$$1 + 2 + 3 + 4 + 5 + 6 + \dots + (n-3) + (n-2) + (n-1) + n$$

Plan:

n

n

n

$1 + 2 + 3 + \dots \quad \dots + (n-2) + (n-1) + n$

each pair $= n$

$\frac{n}{2}$ pairs

$$n \times \frac{n}{2} = \frac{n^2}{2}$$

$= 50 \times 50$

$= 5050$

$$1 + 2 + 3 + \dots + n = \frac{n}{2}(n+1) \Rightarrow \frac{n^2}{2} \approx n^2$$

$\Rightarrow$ Big O $\Rightarrow$ Upper Bound (____)

$$f = 1 + \underbrace{2 + 3 + 4 + 5 + 6 + \dots + (n-3) + (n-2) + (n-1) + n}$$

$$f < n + n + n + n + \dots + n \qquad + n$$

$$f < n^2 \Rightarrow O(n^2)$$

n^2 ✓

Big Ω $\Rightarrow$ lower Bound (____)

$$= \boxed{1 + 2 + 3 \overset{2}{+} 4 + \dots + n}$$

$$f > 1 + 1 + 1\ 1 + \dots + 1 \Rightarrow 1 \cdot n = n$$

$$f \Rightarrow \boxed{\frac{n}{2} + \left(\frac{n}{2}+1\right) + \left(\frac{n}{2}+2\right) + \cdots + \left(\frac{n}{2}+\frac{n}{2}\right)}$$

$$= \frac{n}{2} + \frac{n}{2} + \frac{n}{2} + \cdots + \frac{n}{2} \Rightarrow \left(\frac{n}{2}\right) \text{ terms} = \frac{n^2}{4}$$

Big Θ $\Rightarrow$ Tight **Bound**

$$d\,g \leqslant f \leqslant c\,g$$

$$f \Rightarrow \Theta(g)$$

$$\Theta(n^2$$

$$\left.\begin{array}{l} \lim\limits_{n\to\infty} f(n) \Rightarrow A \\ \hline \lim\limits_{n\to\infty} g(n) \Rightarrow B \end{array}\right\} \Rightarrow O$$

$$\lim_{n\to\infty} \frac{3n^2 + 7n + 5}{n^2} = \; A \;(= \infty)$$

$$\left\{\begin{array}{l} f_1(i_1, i_2, \ldots, i_m) \\ f_2(i_1, i_2, \ldots, i_m) \\ \vdots \end{array}\right.$$

$$\boxed{\begin{array}{l} BL \qquad) \\ \{ \\ \quad f(1,1,\ldots,n) \\ \quad f(1,1,\ldots,n) \\ \quad \vdots \\ \quad f(k,1,\ldots,n) \\ \} \end{array}}$$

$$\underbrace{\left[\begin{array}{l} f_1(i_1, i_2, \ldots, i_m) \\ f_2(i_1, i_2, \ldots, i_m) \\ \vdots \\ f_k(i_1, i_2, \ldots, i_m) \\ \vdots \end{array}\right] \left\{\begin{array}{l} \text{for } (i_1\, 1 ---- n) \\ \qquad = \\ \text{for } (i_1\, 1 ---- n) \\ \qquad = \\ \text{for } (i_1\, 1 ---- n) \\ \qquad = \\ \text{for } (\_\_\_\_\_\_\_\_) \end{array}\right.}$$

$$\boxed{1+2+3+4+\dots+n}$$

U.B. $\underbrace{n+n+n+\dots+n}_{n \text{ times}} \Rightarrow n \cdot n = n^2$

L.B. $1+1+1+\dots+1 = n \rightarrow n \leqslant A \leqslant n^2$

If both (L.B) and (U.B)

$= 1+2+3+\dots+n \Rightarrow$ tight bound :)

$$\frac{1}{4}n^2 \leqslant 1+2+\dots+n \leqslant n^2$$

$$\boxed{\underbrace{\frac{n}{10}(n-1)(2n-1)}_{\geqslant \frac{n}{10}\cdot n^3}+\dots+\underbrace{\left(\frac{n}{2}+\frac{n^2}{10}\right)}_{< \frac{n}{2}\cdot n^2 = \frac{9n}{10}n^2}}$$

$$A > B > C$$

$$\Rightarrow A > C$$

$A \frac{A}{\frac{B}{C}}$

1 ______ n
2 ______ $2n$
3 ______ $3n$ $\Rightarrow \Omega(n^2)$
4 ______ $4n$
. ______ $15n$
. ______ n^2 $\leqslant 9n^2$ $O(n^2)$
. ______ $(n-1)n$
n ______ n

$$O(n^2)$$

$$n^2 + \quad \dots\dots \quad n^2 + n^2 + n^2$$

$$1+2+3+4+\underbrace{5+\dots+n\,(\text{terms})}+n^2$$

$$\Rightarrow O(n^2 \cdot n) - O(n^2)$$

$$(1+2+3+4+\dots+n)\cdot\frac{n^2}{2} + \left(\frac{n^2}{2}\cdot 1\right) + \left(\frac{n^2}{2}\cdot 2\right) + \dots + \left(\frac{n^2}{2}+\frac{n^2}{2}\right)$$

$$\dots\dots + \left(\frac{n^2}{2}\cdot\frac{n^2}{2}\right) + \left(\frac{n^2}{2}\cdot\left(\frac{n^2}{2}+1\right)\right) + \left(\frac{n^2}{2}\cdot\left(\frac{n^2}{2}+2\right)\right) + \dots + \left(\frac{n^2}{2}\cdot\left(\frac{n^2}{2}+\frac{n^2}{2}\right)\right)$$

$$\geqslant \frac{n^2}{2}\left(\frac{n^2}{2}\right) = \frac{n^4}{4} \quad \boxed{\Omega(n^4)} \quad O(n^4)$$

1- $f(i_1, i_2, \dots, i_m, i_n) \longrightarrow i_1^{k_1} + i_2^{k_2} + \dots + i_n^{k_n} \Rightarrow n^k$

$n = \max\{i_1, i_2, \dots, i_n\}$

2- $f(i_1, i_2, \dots, i_n) \rightarrow \dfrac{(i_1^{k_1} + i_2^{k_2} + \dots + i_n^{k_n}}{\boxed{1+2+3+\dots+n}} = O(n^k)$

$\left(\text{since } 1+2+\dots+n = \frac{n(n+1)}{2} = \Theta(n^2)\right)$

$$\boxed{\frac{n^k}{n^2} = O(n^{k-2})}$$

$$n^2 + n^4 \Rightarrow O(n^4)$$

$$\left[\begin{array}{l} \text{for } (i_1, i_2, \dots, i_n) \\ \text{for } (j_1, j_2, \dots, j_m) \end{array}\right] \begin{array}{l} \longrightarrow 1+1+1+\dots+1 = n \\ \longrightarrow \left(\frac{n(n+1)}{2}\right) + \underbrace{\left(\frac{n(n+1)}{2}\right) + \dots + \frac{n^m}{m}}_{m \text{ times}} \end{array}$$

$$n + n^2 \Rightarrow \Theta(n^2) \rightarrow n^2$$

$k = \sqrt{n}$

$$\left[\begin{array}{l} \text{for } (i_1, i_2 = k : i+1) \\ \text{for } (j_1, j_2 = k : j+1) \end{array}\right] \begin{array}{l} \longrightarrow \sqrt{n} \\ \longrightarrow \underbrace{\sqrt{n} \times \sqrt{n} \times \sqrt{n} \times \dots \times \sqrt{n}}_{\sqrt{n} \text{ times}} = n^{\frac{\sqrt{n}}{2}} \end{array}$$

$\text{for } (i_1, i_2 = k : i+1) \Rightarrow 1+1+\dots+1 \ (\sqrt{n})$

$\text{for } (j_1, j_2 = 0 : j+1) \Rightarrow \boxed{1+2+3+\dots+\sqrt{n})}$

$$\boxed{\sqrt{n} + \Theta(n) \Rightarrow \Theta(n)}$$

$1+2+3+\dots+n \Rightarrow n^2$

$1+2+3+\dots+\sqrt{n} \Rightarrow \frac{(\sqrt{n})(\sqrt{n}+1)}{2} = n$

$$n^2 + n^3 + n^4 + n^1 + \dots + n^k \Rightarrow n^k$$

$$1 + 2 + 3 + 4 + \dots + n^2$$

$$1+2+3+4+5+6+7+8+9+\dots+n^2$$

$$= \Theta(n^2) \text{ terms} \times n^2 = \Theta(n^4)$$